ON THE KINDS OF MOTION IN THE GRAVITATIONAL ESCAPE OF LOOSE MATERIALS

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The kinematics of the escape process of granular materials from vessels with escape holes located in a horizontal flat bottom is investigated. By using moving pictures, the boundaries of the zones of the different state and the motion of the loose material are determined.

Hoppers and silos are used to gather and store loose materials. Their constructions, although diverse, are not different in principle. Hoppers and silos are distinguished mainly by their dimensions, principally the ratio of the height to the diameter.

On the basis of assertions by many scientists about the independence of the escape efficiency from the height of the column of loose material in the vessel, it can be assumed that the character of the motion

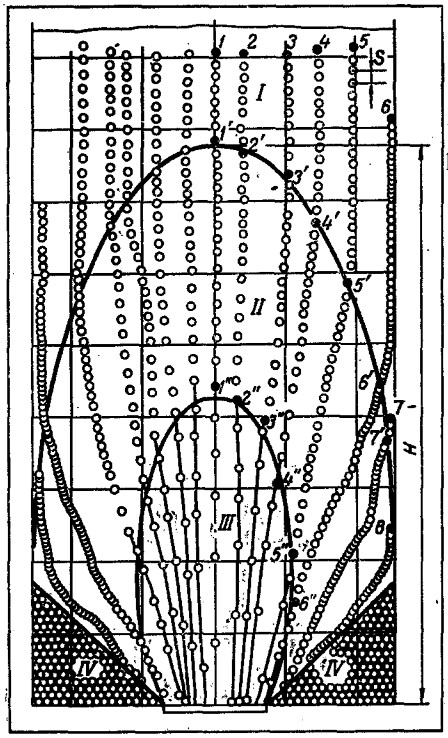

Fig. 1

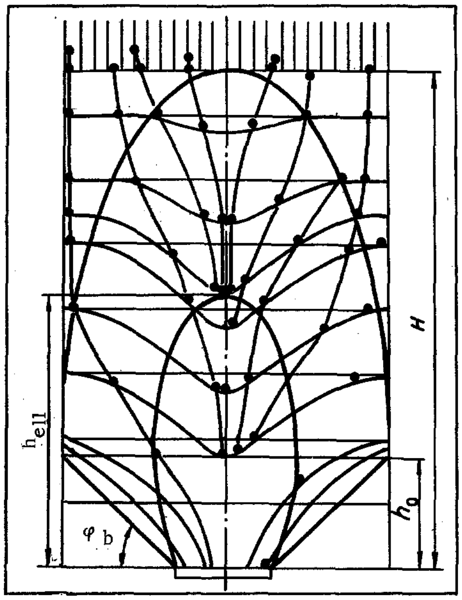

Fig. 2

Fig. 1. Particle trajectories and zones of a different state during the escape of grain (corn): I) coupled motion zone; II) transition zone from coupled to uncoupled motion; III) zone of intensive motion; IV) slopes of fixed material; S) path traversed by particles after 10 frames of the moving picture film at a 64 frames/sec rate.

Fig. 2. Scheme of funnel formation on free surface of loose material.

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TABLE 1. Physicomechanical Properties of the Loose Materials

Loose material	Bulk weight γ_b, g/liter	Volume mass in hopper γ_h, kg/m³	Specific gravity γ_{sp}, kg/m³	Moisture W, %	Slope of repose		Particle dimensions		
					pouring $\varphi°$	shattering $\varphi_b°$	length a'	width b'	thickness c'
Peas	788	810-830	13,0	12,7	18,5	37,5	6,87	6,05	5,63
Corn	734-752	785-795	12,0	13,0-11,65	32,7-34,1	48,4	8,40	7,67	4,60
Millet	712	730	10,6	12,3-11,7	28,0-29,0	30,15	2,93	2,31	1,89
Wheat	757	785-801	12,2	13,2-11,6	32,0-34,0	38,3	6,67	3,03	2,86

TABLE 2. Experimental and Computed Values of the Dome Heights H and h_1 in a Cylindrical Hopper

d, mm	H, cm			$h_0 = \frac{D-d}{2} \mathrm{tg}\varphi_b$, cm	h_1, cm		
	expt.	from (3)	error, %		experimental from (1)	calc. from (2)	error, %
Corn γ_b=790 kg/m³, φ_b=43,4°							
40	36,6	36,4	+0,55	9,9	26,7		+0,75
50	38,3	36,0	+6,4	9,5	28,8		+8,7
60	33,8	35,5	—4,8	9,0	24,8	26,5	—6,4
70	33,9	35,0	—3,15	8,5	25,4		—4,15
90	30,8	34,1	—9,65	7,6	23,2		—12,5
Wheat γ_b=792 kg/m³, φ_b=38,3°							
30	38,0	40,3	—5,7	8,7	29,3		—7,3
40	37,8	39,9	—4,8	8,3	29,5		—6,6
50	37,1	39,5	—6,1	7,9	29,2	31,6	—7,6
60	35,3	39,1	—9,7	7,5	27,8		—12,0
70	36,8	38,7	—4,8	7,1	29,7		—6,0

will be analogous during escape from both kinds of vessels under equivalent conditions (identical vessel shapes, escape holes, the same kind of loose material, etc.).

The author conducted an experimental investigation of the grain escape process (peas, corn, millet, and wheat) from flat-bottomed vessels of cylindrical, semicylindrical, and prismatic shape with $D = A_c = 250$ mm diameter and 800 and 1400 mm altitude. The escape holes of circular, semicircular, square and rectangular shape had 15-200 mm dimensions.

The fundamental indices of the physicomechanical properties of the loose materials under investigation are presented in Table 1.

A moving picture of the escape process was taken. The escape holes of appropriate shapes were hence disposed so that one or two of their rectilinear edges adjoined the flat glass walls [1]. For convenience in the measurements, a 5×5 cm² dimensional grid was superposed on the transparent wall and part of the grain loaded in the bunker was colored with black india ink in order to facilitate observation of the particle motion. The moving picture method and the test results are described in detail in [2].

The trajectories and velocities of individual particles were determined by using the moving pictures. The measurements were conducted with high repetitivity (from 30-80). The test error was less than 1%.

Shown in Fig. 1 as an illustration are the successive locations of corn kernels after 10 frames of moving picture film at a 64 frames/sec rate. The escape was through a 70×70 mm² square hole.

An investigation showed that depending on the state of the loose material in the vessel, four characteristic zones can be distinguished during the escape (Fig. 1).

Zone I. The loose material moves as a single whole without relative displacements of the particles; the trajectories of all the grains are vertical and the velocities in any horizontal section are identical and do not vary over the altitude of the vessel. We call such motion coupled by using the term proposed by P. N. Platonov in [4]. The exception is just the 1-2 grain layer near the wall which moves at a somewhat slower velocity because of the decelerating effect of the vessel walls.

The boundaries of this zone are: top – the free surface of the loose material, side – the vessel walls, bottom – some provisional curved surface in the shape of a dome resting on the slopes of fixed material (zone IV) where they adjoin the hopper walls. A vertical section of this plane through the hole axis has an almost parabolic shape in a cylindrical vessel with a circular hole and elliptical shape in a prismatic vessel with square hole.

It has been established by experiments that the shape and size of the curve mentioned are determined by the physicomechanical properties of the loose material, the shape and cross-section of the vessel, and are independent of the hole dimensions (Table 2).

Zone II is bounded from above and below by curved surfaces. The character of the motion can here be elucidated by tracking the displacements of particles 1-8 (Fig. 1). When the trajectory reaches the upper

TABLE 3. Experimental Values of the Height of the Initial Formation of a Funnel H and Intensive Motion Ellipsoid h_{ell} in a Prismatic Hopper with Square Hole

Quantity measured	Size of side of the hole A, mm				
	30	50	70	90	110
H, cm	40,5	39,4	38,5	37,5	36,4
h_{ell}, cm	12	18	21	24	27

boundary of this zone, the majority of grains starts to be deflected towards the hole axis and the more strongly, the farther away they are. Simultaneously, a gradual growth in velocity occurs. The layered packing of the grains is spoiled. The particles in any horizontal section are displaced relative to each other. Such motion is called uncoupled in [4].

The particle velocity increments in this zone are still small; hence, it can be considered as the transition zone between coupled and uncoupled motion.

Zone III. If the hole is centrally located it has an outline similar to an ellipsoid of revolution, whose major semi-axis coincides in direction with the hole axis, and whose minor axis is approximately at the level of the upper boundary of the slopes of the fixed material.

In the third zone, as in the second, the motion is uncoupled but the change in particle velocity as they progress towards the hole is more rapid. All the particles here move with acceleration while their velocity in the lower part of the transition zone diminishes.

The direction of the trajectory again changes somewhat upon entry into zone III. The lesser drags due to gravity because of disintegration of the material make them closer to a vertical direction.

Zone IV is the fixed material. This zone is a cone whose generators are inclined at the shattering angle of repose φ_b to the horizon in a cylindrical hopper with flat bottom and circular hole. The boundary surface has a more complex shape in a prismatic hopper with square hole; however, the particle declinations are approximately the same.

When the free surface of loose material drops down to the point 1' a funnel starts to be formed there. This process terminates near the point 1" at a distance h_{ell} from the plane of the escape hole (Figs. 1 and 2). The appearance of a funnel at the free surface of the material is a result of the dissimilar growth in the particle velocities in the horizontal sections of the stream during transition from the coupled to uncoupled motion. The height of the beginning of funnel formation H, which is simultaneously the height of the upper point of the transition boundary from coupled to uncoupled motion, was determined experimentally as follows for a hole in the center of the bottom: a short metal ruler was set on the leveled surface of loose material and the descent was observed. As soon as a gap appeared at the center between the ruler and the surface of the material, the escape was cut off and H was measured. The mean values of the test results are presented in Table 2.

The height of the dome can be found from the expression

$$h_1 = H - h_0. \quad (1)$$

The results of a computation are also given in Table 2. With satisfactory accuracy, the dependence

$$h_1 = \frac{D}{\mathrm{tg}\varphi_b} \quad (2)$$

satisfies the values of h_1 found in this manner for corn and wheat for a circular hole in the center of the bottom of a cylindrical hopper.

The total height of the dome over the plane of the escape hole will be

$$H = \frac{D-d}{2}\,\mathrm{tg}\varphi_b + \frac{D}{\mathrm{tg}\varphi_b}. \quad (3)$$

When the hole was at the glass wall, H was determined by using the moving picture film. The data for corn are presented in Table 3 for a hole in the wall. Given there are values of h_{ell}, the maximum height of the intensive motion ellipsoid, which equals the height of the loose material up to the time of termination of funnel formation.

It is seen from Table 3 that the height H is reduced as the size of the hole increases, while h_{ell} grows, i.e., the transition zone is contracted.

The dependence of h_{ell} on the size of the hole is expressed by the empirical dependence

$$h_{ell} = C\sqrt{A}. \tag{4}$$

where C = 16 for corn.

The test results do not contradict the hypothesis held by many scientists but not yet accorded total recognition, about the dynamic domes originating in the mass of loose material during escape [5-10, etc.]. By taking this hypothesis we attempt to explain some phenomena we observed during investigations.

If the escape hole is sufficiently small, then after the first portion of grain has fallen, a dome closes over it and the escape ceases. The static dome is formed for a maximum density in the vessel. The material below it receives the greatest compression during filling. As is known, the diverse reasons causing compression (vibrations, etc.) raise the tendency to the formation of stable domes. By the assertion of many researchers, the static dome is paraboloidal in shape. Its greatest transverse dimension equals the hole dimension, and the altitude is determined by means of the known formulas of M. M. Protod'yakonov, etc.

If the escape does not cease, then a downward displacement of the mass lying above occurs right after dumping of the first portion of grain. The velocity of particle motion near the hole is greater than at the top of the vessel, hence disintegration of the material is inevitable here.

A diminution in the density results in attenuation of the friction bonds between the particles and closure of the dome at the same height as the static will not occur. It can be formed only there where there is sufficient density and friction coupling between adjacent particles, i.e., considerably higher.

An increase in the dome height reduces its stability. Having originated, it ruptures immediately under the pressure of the layers lying above. This process should occur continuously during escape. The most favorable place for this is visibly the transition zone II which can only very provisionally be called a dome, in our opinion. Indeed, its outline recalls the latter but we did not succeed in detecting any jump in velocity during passage through the "dome." In all probability zone II can be considered as a set of surfaces on which points of particle contact are located as they move, which according to the theory of dynamic domes will ensure pressure transmission to the vessel walls.

The deflection of the particle trajectories in the lower part of the transition zone to the side of the hole axis should apparently entail some increase in density in the section of the intensive motion zone directly above the escape hole. Its least value should be expected in the horizontal section of the flow at a distance h_0 from the bottom of the vessel (Fig. 2).

The change in density might be the reason that the outline of the intensive motion zone boundary is distorted as compared with the shape of the static dome. As experiments have shown, it is closer to an ellipsoid than a paraboloid.

The majority of investigators concerned with an experimental study of the motion of loose materials in hoppers and silos also note two fundamental escape modes, or two kinds of motion, but characterize them with dissimilar criteria and different terminology: escape by a funnel and a whole column [11], normal and hydraulic [12, 13], first and second escape modes [14], coupled and uncoupled [4], primary and secondary motion [15], etc. Such a manifold of terminology to denote the same phenomenon is not expedient and makes the use of literature on the mechanics of loose materials difficult.

The process of emptying a hopper or silo can be understood as escape in the broad sense of the word. Two kinds of motion are simultaneously observed: 1) with identical and constant particle velocity in any stream cross-section and vertical trajectories; and 2) with different, continuously varying particle velocities due to their relative displacement, and intersecting trajectories. Under definite conditions, for example, for a height of the loose material in the vessel less than in Fig. 2, there may not be the first of the listed kinds of motion, coupled motion, in a conical hopper, etc.

In the narrow sense, escape can be called the motion of the material through the escape hole. As the investigations of many authors and our experiments show the loose material always passes through the hole with uncoupled motion during free escape.

Therefore, the kind of motion of the loose material should be taken as one of the principal characteristics of the escape process.

The majority of terms presented above do not reflect the differences in the kinds of motion during escape, and are provisional. The terms "normal" and "hydraulic" escape are used more often than others in domestic literature. But their characteristics are not only diverse but also sometimes contradictory among different authors [7, 12, 13]. Taking account of the above, the fundamental criterion of the escape mode as a complex concept can be the presence of two or one kind of (uncoupled) motions during escape. The terms "coupled" and "uncoupled" motion from [4], which are also used herein, express the kinematic side of the phenomenon most exactly, in our opinion. The motion during which the couplings at the contact points of particles of a loose body are not spoiled and the velocity gradient along the stream section is zero is called coupled in [4]. Uncoupled motion is characterized by the fact that the particles in the stream have relative displacements, their couplings are spoiled, and the velocity gradient along the stream cross section is different from zero.

NOTATION

A	is the size of the side of a square hole;
A_c	is the size of the side of the cross section of a prismatic vessel;
a'	is the greatest dimension of a loose material particle (length);
b'	is the width of a loose material particle;
C	is the experimental coefficient dependent on the physicomechanical properties of the loose material;
c'	is the thickness of a loose material particle;
D	is the diameter of the cylindrical vessel;
d	is the diameter of the escape hole;
H	is the height of the beginning of funnel formation on the free surface of loose material;
h_0	is the height of slopes of fixed material;
h_1	is the dome height;
h_{ell}	is the height of the upper point of the intensive motion ellipsoid above the escape hole plane;
S	is the displacement of a loose material particle per unit time;
W	is the humidity;
γ_h	is the volume mass of loose material in the hopper;
γ_b	is the "bulk weight" (nature of the grain);
γ_{sp}	is the specific gravity of the loose material;
φ	is the angle of repose of filling;
φ_b	is the angle of repose of shattering.

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